Predicting the Electoral Effects of Mandatory District Compactness

By Micah Altman[1]

[1]Division of Humanities and Social Sciences 228-77, California Institute of Technology, Pasadena CA 91125.

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Abstract

Recent Supreme Court decisions have caused a renewed interest in controlling the redistricting process. The courts have paid particular attention to formal criteria that can be used to regulate and evaluate the creation of districts. One of the most important and controversial of these criteria is geographical compactness, which is a measure of the “ugliness”, or irregularity, of a district's shape.

In this paper, I examine the electoral effects of compactness standards on political representation when some political groups are geographically concentrated. I examine the neutrality of compactness standards, and the ability of such standards to prevent gerrymandering by treating redistricting formally as a combinatorial optimization problem. Since these problems cannot, in general, be solved exactly, I use Monte-Carlo techniques, simulated annealing, genetic algorithms, and other simulation techniques to solve them approximately. These simulations reveal that compactness standards do constrain electoral manipulation, but that they are not politically neutral.

1. Redistricting Rules and Bias

From the time that geographical districting was first used in the United States, irregularly shaped districts have been a subject of popular attention, evoking criticism from the press, but little action from the courts. (Griffith 1974) In recent years, however, the courts have begun to scrutinize district lines. To aid the courts in this scrutiny, the academic community has developed formal methods of measuring the geographic compactness of districts.

Formal compactness measures are mathematical functions that describe irregularities in district shape or population distribution. In other words, these are measurements used to detect "ugly" districts, or formulae used to generate "pretty" ones. Most proponents of compactness measures offer them as a means to prevent electoral manipulation.

The Supreme Court has used informal notions of geographical compactness in a number of its recent decisions, and this trend is likely to continue. In the first section of this paper I review these recent court decisions. I shall argue that these decisions are likely to have severe political consequences — especially for political & ethnic minorities.

Although the academic community has proposed more than thirty compactness standards, we remain unsure of how such standards will change politics. Proponents argue that compactness measures will "cure" gerrymandering, but they offer little empirical or theoretical evidence to support such claims. In the second section, I discuss the questions asked by previous writers about compactness: Are compactness standards politically neutral? Will such standards prevent gerrymandering?

This paper provides answers to these questions. Compactness standards will reduce gerrymandering by severely limiting the choices available to a district planner. Unfortunately, compact districts will disproportionately affect geographically concentrated political groups. Geographical compactness standards are not politically neutral.

In sections three and four, I use a three stage model to examine the electoral effects of formal district criteria. First, I equate the task of drawing compact districts to an optimization problem, which I solve with combinatorial optimization techniques. By treating the problem in this way I am able to draw thousands of compact district plans that are free from personal bias. Second, I generate many possible population maps, according to different clustering functions. With these maps I abstract away the eccentricities of any one local area, and focus on the electoral effects of general population characteristics on redistricting. Third, I examine the electoral outcomes that would be most likely under each plan, and I relate these outcomes to the geographical compactness of the district.

1.1 The Increasing Legal Importance of Compactness in Redistricting

District compactness has gradually become more important since the Supreme Court entered the "political thicket" of redistricting. In the first two and a half decades after its entry, the court mentioned district geography, but did not use compactness as a decisive factor.[2] Even

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[2]Compactness did not play a primary role in redistricting before *Gingles* and *Shaw*, but it has been referred to in several other cases:

• In *Gomillion v. Lightfoot* (1960), the court's first entry into "the political thicket", the court struck down Alabama's attempt to redraw the boundaries of the city of Tuskegee. While compactness was not cited as a primary consideration, the court's reference to the proposed district as "an uncouth 28-sided figure", indicates that noncompactness was used as an indication of illegal behavior.

• In *Connor v. Finch* (1977), the court rejected Mississippi's district plan on grounds other than compactness, but then included compactness as a standard to be met when Mississippi correctly redrew those plans.

• In *Davis v. Bandemer* (1986), Justice Stevens, dissenting , states that noncompactness is indicative of a gerrymander.

• In *Karcher v. Daggett* (1983) Justice Stevens, concurring, states that "drastic departures from compactness are a signal that something may be amiss".

now, the Court neither recognizes a Constitutional right to compactness (*Wells v. Rockefeller*, 1968), nor specifies how to measure it. Recently, however, compactness has played a larger role in redistricting cases. Three of these cases particularly emphasize compactness:

• In *Thornburg v. Gingles* (1986) the court decided that, under the 1982 amendments to the Voting Rights Act, in order to demonstrate that at an-large plan has a discriminatory effect, one must show that at least one minority opportunity district that is geographically compact can be created within the bounds of the multi-member district.

• In *Growe v. Emison* (1993), the court applied the standards in *Gingles* to single member districts. The court ruled that these standards, compactness among them, were minimum criteria for the creation majority-minority single member districts

• In Shaw *v. Reno (*1993*)*, the court allowed a challenge to North Carolina's redistricting plan to proceed on the basis that the ill-compactness of the districts indicated a racial gerrymander. Justice O'Connor’s words in this case especially emphasized the role district shape played in the decision: “we believe that reapportionment is one area in which appearances do matter.”

Shaw significantly affected subsequent redistricting cases. It played a significant role in the Louisiana district court’s decision to reject the offered districting plan in *Hays v. Louisiana* (1993). In addition, the discussion of *Shaw* in Justice Kennedy’s concurrence in *Johnson v. DeGrandy* (1994) and Justice Thomas’s concurrence in *Holder v. Hall* (1994), underline the importance of the case and the issues it raises.

The court’s holding in the most recent redistricting case, *Miller v. Johnson* (1995) magnifies the importance of district compactness. The court, by adopting a strict scrutiny rule for majority-minority districts, increases the pressure on states to follow compactness standards — along with the other “traditional districting criteria” mentioned in *Shaw*.

1.2 Compactness, Manipulation, and Bias

District planners now may want to go in the direction set by the courts and use "traditional districting criteria. But how will they tell whether their plans are compact? If there existed well-known, effective, fair compactness standards, the district planner's job would be simple. Unfortunately, more than thirty compactness measures have been proposed, and none of these measures has been rigorously examined. Scholars disagree about the consistency of these measures, their effectiveness in preventing electoral manipulation, and their neutrality.

These last two issues have received particular attention. Political scientists disagree vehemently over the effectiveness and neutrality of compactness measures. Claims about compactness often sharply conflict — compactness has been called both a panacea against electoral manipulation (Polsby and Popper 1991; Stern 1974) and a "Republican Trojan Horse" aimed at systematically reducing minority and Democratic representation. (Lowenstein and Steinberg 1985).

Current empirical research does not help district planners to determine which compactness measures to use, which are effective, and which are neutral. It is to this research we now turn.

2. Current Research on the Effects of Compactness Standards

Empirical research on compactness has been limited, for the most part, to descriptive measurements of a few cases. A number of studies take individual compactness measures and apply them to a small number of district plans. (Hofeller and Grofman 1990; Niemi and Wilkerson 1990; Reock 1961) Studies have also compared several measures of compactness (Flaherty and Crumplin 1992; Niemi, et. Al. 1991). In one of the larger empirical studies Pildes and Niemi (1993) calculate several measures of compactness for congressional districts from 1980 and 1990, and examines changes in district compactness over that period.

Most of the debate over compactness standards has been informal. Compactness is occasionally seen as having intrinsic value: There are some claims that compactness promotes communities of interest,[3] or that it reduces the cost of electoral campaigns and the costs of representative-constituency communication.[4] Most of the debate over compactness, however, centers on whether or not we can use compactness standards as an instrument to detect and prevent gerrymanders.

While there are few studies that systematically address the questions of the effectiveness and neutrality of compactness standards, many claims about have been made. These claims are easily summarized:

2.1 Pro-Compactness Claims

• Compactness is a neutral standard that in combination with a few other criteria, such as contiguity and equal population standards, will make gerrymandering effectively impossible. (Stern 1974; Wells 1982) — "(compactness standards will) make the gerrymanderer's life a living hell." (Polsby & Popper, 1991, pg. 353), (Polsby and Popper 1991)

• While not sufficient in and of itself, compactness is a useful, neutral, and objective criterion for limiting gerrymanders. (Morrill 1990) But, "Geographers, who have a particular concern for territorial measuremen, know not to expect too much from a compactness criterion." (Morrill, 1987, pg. 249) (Morrill 1987)

• Compactness standards are a means of repairing the effects of systematic Democratic gerrymandering.(Congressional Quarterly Staff 1992; Congressional Quarterly Staff 1993)

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[3]If community of interest is the goal, compactness is an unlikely instrument for promoting it (Cain 1984) since communities of interest may not be geographically compact.

[4]This is probably not as much of a consideration given today's communication's technology. It has however, been used as an argument against at-large elections, since campaign costs are likely to be most burdensome on minorities. (Davidson 1984)

• Compactness standards will result in increased minority representation:

> It (territorial compactness) has a practical advantage. Any territorially compact minority will tend to gain — in the sense of "electoral success," the VRA's basic criterion of "opportunity — from an antigerrymandering principle. So, we strongly suspect, would the black population of North Carolina, geographically dispersed though it is... (Polby and Popper 1993, 681)

2.2 Anti-Compactness Claims

• Compactness measurements are not sufficiently restrictive to prevent electoral manipulation in most cases — "(compactness) provides benefits more illusory than real" (Musgrove 1977, 56)

• Compactness standards put severe limits on some types of gerrymandering, but may also be used to "pack" districts to the disadvantage of an opponent. (Hacker 1964)

• Deviations from compactness cannot be used to discover gerrymandering which is used to (dis)advantage a group, but may be useful as a signal of incumbent gerrymandering. (Grofman 1985)

• Ill-compactness is a warning signal that requires justification, but "compactness alone does not make a redistricting plan good." (Niemi et al., 1991, 1177)

• Compactness measurements are only useful if used to force an explanation for odd shaped districts. If they are used as rigid requirements, they will cause a harmful shift of attention from politics to mere geography. (Dixon 1968)

• Compactness measures do not capture manipulation consistently, and they will produce more subtle and invidious gerrymandering: "This reliance on formulas has the semblence, but not the substance, of justice." (Young 1987, 113)

• Compactness is of little intrinsic value and conflicts with good government criteria such as minority vote protection, district competitiveness, fair "swing ratios", respect for political and geographic subdivisions, respect for communities of interest, and other factors of intrinsic value. (Aleinikoff and Isacharoff 1993; Cain 1984; Lijphart 1989; Mayhew 1971) Compactness standards are only weak proxies for that which is really important — civic inclusion (Karlan 1989)

• Compactness standards are inherently biased against urban dwellers, and will systematically hurt Democrats and minorities:“By and large, however, the introduction of a compactness rule significantly tilts the game in favor of the Republicans.” (Lowenstein 1985, 25)

Proponents of compactness claim that the compactness measures are effective at preventing manipulation and are inherently neutral. Many opponents of these measures disagree, claiming that the application of these measures will be ineffective in the prevention of gerrymanders.

Opponents of compactness standards also claim that such standards will split political groups that are geographically concentrated— reducing the representation of such groups. Because minority populations are disproportionately concentrated in large cities such fragmentation would have serious consequences for minority representation.

The debate will turn on the answers to three essential questions:

• Can compactness be measured consistently and sensibly? And if so, which compactness measures should we use?

• Can mandatory compactness standards prevent gerrymandering?

• Are compactness standards neutral — or will they systematically benefit certain political groups?[5]

[5]Aside from any affect the prevention of gerrymandering might have, of course.

The best way to determine whether compactness measures are consistent is to analyze the formal properties of those measures. Two authors have performed such analyses (Altman 1995; Young 1988), and I will not repeat this work here. This paper will focus on the latter questions of effectiveness and neutrality.

Will compactness standards be effective? Will these standards disproportionately affect some political groups? To evaluate these claims, in this paper I pay special attention to the interaction of compactness rules and population characteristics. In the next section, I shall show how geographically concentrated political groups can benefit from compactness standards.

3. Predicting the Effect of Compactness Standards

Compactness standards are based geography; to predict their electoral effects we must understand how the geography of districts interacts with the geography of political groups. In this section I show how we can use simulations to predict these interactions.

This section is divided into three parts. In the first part of this section, I use a simple example to illustrate the relationship between population patterns, compactness and representation. In the second part, I argue that simulation techniques provide uniquely effective ways of analyzing this relationship. In the third part, I discuss the simulation model in detail.

3.1 Population Distribution and Compactness: a Small Example

To see how district geography can interact with political geography consider a hypothetical state shaped as a square. This square state is inhabited by two factions with distinct policy preferences, the “Republicans” and the “Democrats. Members of these factions live in each of the state’s thirty-six equally-sized population blocs.

The political structure of this hypothetical state is as simple as its population. The district map for this state contains four districts, each of which is made up of nine indivisible population blocs. Each district elects a member of the legislature by majority rule (ties are decided by a coin toss).

"Republicans" compose one quarter of the population. Furthermore, the "Republican" population is not uniformly distributed, but instead is concentrated compactly[6] in one area. Each population bloc is homogenous and is entirely occupied by either "Republicans" or "Democrats".

Two possible population distributions, distribution 'A' and distribution 'B', are shown below:

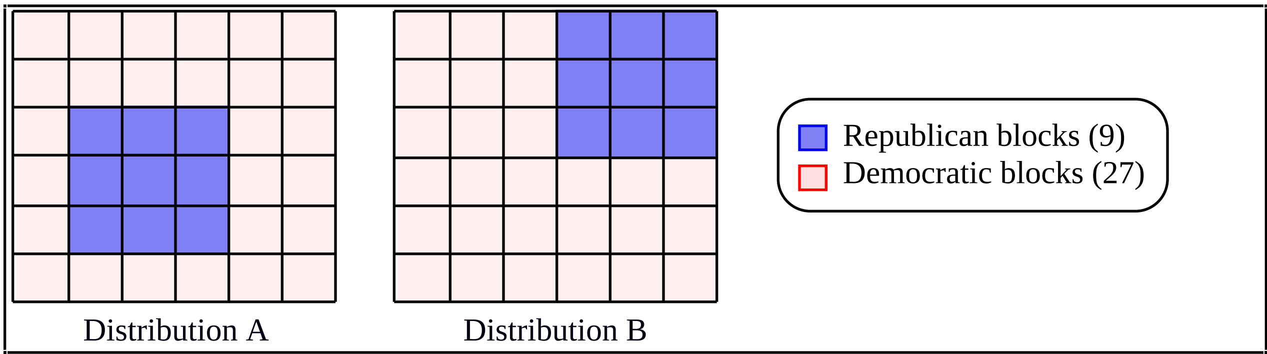

Figure 1. In a hypothetical state with a compact minority population, two alternate population patterns are shown

How do compactness standards affect the drawing of districts in this tiny state? In this simple state, there is a unique, equipopulous, maximally compact district plan. The maximally compact plan is shown below:

[6]For this example a very simple definition of compactness is adequate. District compactness will be taken to be $\frac{\text{min district diameter}}{\text{max district diameter}}$, and plan compactness will be taken to me mean district compactness. While a more sophisticated measure is desirable in general, for this simple case, most definitions of compactness coincide on both the maximally compact plan, and the maximally compact district shape.

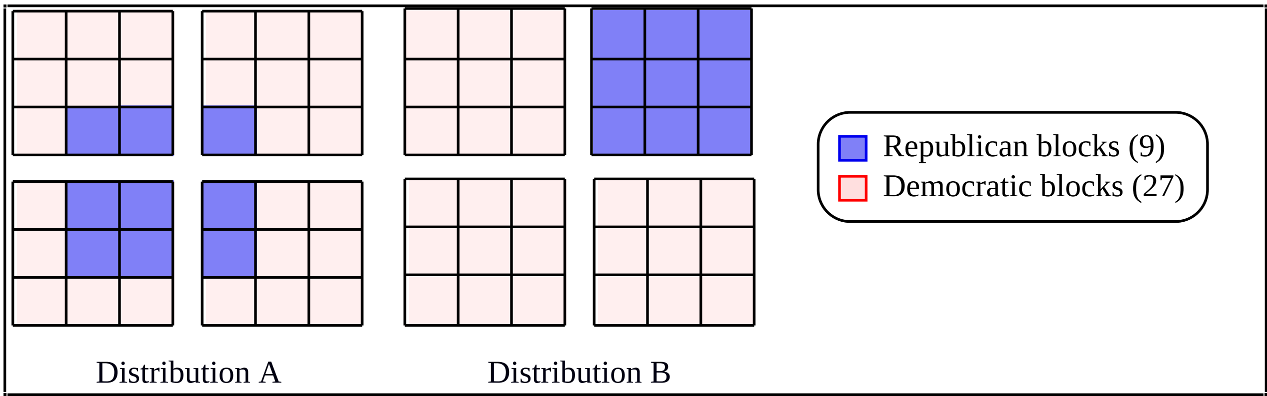

Figure 2: The unique, maximally compact district plan for the hypothetical case above. Political consequences of this plan depend on the population distribution. Under distribution ‘A’, Democrats are a majority in every district, while under distribution ‘B’, Republicans control one district.

The political results of this district plan depend, however, upon the population patterns in the state. Both population patterns ‘A’ and ‘B’ are quite similar — in each, the minority is maximally compact and makes up one quarter of the population. Under population distribution ‘A’, however, the legislature is entirely controlled by Democrats, whereas under population distribution ‘B’, Republicans capture one seat.

In case ‘B’ above, mandatory compactness ensures that the “Republican” faction will be entirely shut out of the legislature. The compactness rule still limits gerrymandering, in the literal sense that it makes it impossible to manipulate the results of an election by changing district lines. However, the rule has a clearly disproportionate effect on different political groups. If someone who knew the population distribution had suggested such a compactness rule, we might suspect them of partisanship.

This is only one case, and this case is designed to illustrate the issues, not to depict reality. All rules have some political effect in every case. We have reason to suspect the neutrality of a rule only if it can be expected to *systematically* benefit a particular political group.[7] It is this issue of systematic effect that I will explore in the rest of this paper.

This example, while it does not show what compactness standards *will* have, shows three effects that compactness can have: Compactness rules can limit the possibility of electoral manipulation; but while these effects may depend systematically on how political groups are geographically distributed, geographically compact groups may not be well represented under a compact plan.

3.2 How can we predict systematic interactions between geographical compactness and political geography?

In this paper, I use simulation to overcome the shortcomings of empirical analysis and to provide direction and structure for future empirical analysis. Three factors have previously hindered an empirical approach.

First, the exact relationship between compactness requirements and gerrymandering is not well understood. No formal models describe the mechanisms by which compactness requirements act to limit gerrymandering. Indeed, the controversy over the effectiveness of these measures indicates our lack of knowledge about this problem.

Second, there are a number of different political and geographical reasons for “ugly” districts. Ill-compact districts may be caused by geographical constraints; by an underlying unevenness in the distribution of population across a state; by attempts to follow “natural” political boundaries; or by political attempts to manipulate lines for the benefit of communities

[7] Actually, the effect in this hypothetical example cannot be said to be systematic, as the probability of a particular population distribution has not been specified.

of interest, racial minorities, political parties, or incumbents. These causes are difficult or impossible to measure, and they may interact in complex and confounding ways — district lines drawn to protect incumbents in one district may be compact, in and of themselves, but may cause a neighboring district, drawn in absence of any political motive, to be ill-shaped. Because of this multitude of factors, it is usually quite difficult to be statistically certain of whether the compactness or ill-compactness of a district reflects a political decision, and if so, who would, in general, benefit from a more compact plan.

Third, the set of suitable empirical data is limited both in breadth and depth. It is only in the presence of equal population requirements that a compactness standard achieves its full effect — when the population of each district is unrestricted, politicians can easily manipulate electoral outcomes without drawing ugly districts. Since equal population requirements have been judicially enforced in the U.S. only since the late 1960's, there are only a small number of plans that are available for our analysis. Even among these plans, the limited range of the independent variables obstructs statistical analysis.[8]

Simulation offers us an opportunity to overcome these limitations, to abstract away from the geographical and political eccentricities of any one plan, and to directly analyze the connections among the shapes of districts, the distribution of political groups, and the outcomes of elections.[9] By creating districts under compactness rules, we can explore the relationship of compactness and districting. We can use simulation to draw districts based only upon

[8]Compactness changes only over a very small part of its possible range. Similarly, minority population percentage and distribution tend to remain relatively stable on the scale of an entire redistricting plan. This is especially true since the states which contain significant minority populations tend to have a relatively large number of districts.

[9]Here, akthough I use simulation at a higher level of abstraction than perhaps they envisioned, I am in rare agreement with Polsby and Popper (1991) : "Enough knotty statistical issues must be overcome that probably the only way to settle this point (the effect of compactness standards) is through empirical analysiss - running thousand of computer models of compact districts and seeing what happens" (pg. 335, fn.)

compactness considerations —with no ulterior motives. Because we can isolate compactness from other political variables, we can disentangle its effects from other confounding effects. In addition, we can use simulation to generate nearly unlimited numbers of plans, which allows patterns in the data that might otherwise have been obscured by statistical noise to emerge clearly.

Finally, we can use simulation as a basis for creating empirically testable hypotheses. In the absence of detailed theoretical models of geographical redistricting, it has been difficult to develop specific, empirically testable hypotheses about compactness. While this simulation purposely does not mimic reality, it points to clear relationships between compactness and population distribution which can be empirically tested.

3.3 The Model

Historically, computers have been frequently used to create redistricting plans — but usually as a tool to assist human planners.[10] (Browdy 1990a). I treat redistricting as a mathematical set-partitioning problem, and use automated districting techniques to generate a series of arbitrarily drawn district plans. I complement these automated districting techniques with general combinatorial optimization algorithms that have been used successfully on similar problems in computer science. This simulation model departs from the standard use of computers in two respects.

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[10]There are some notable exceptions to this usage: Shepard and Jenkins (1970) and Taylor (1973) also use automation procedures to examine a range of districting options, but apply their techniques to the analysis of a particular election, rather than to an election rule. While Engstrom and Wildgen (1977), Taylor and Johnston (1979), and O'Loughlin (1982) argue that automatically-created districts should be used as a benchmark with which to detect gerrymandering.

First, I use the computer not simply to generate districting plans, but to examine how different districting rules constrain the process of drawing plans. Second, I analyze not only the way in which these different rules might affect a single political group, but how they would affect any political group with a particular population distribution.

The idea behind the simulation is simple — we use the simulation to generates "random samples" of district plans under controlled circumstances. We create these samples in three steps:

• First, we specify a real or artificial geography.

• Second, we add populations to this geography, either according to a specified mathematical function or method, or by using census data.

• Third, we specify the criteria that a district plan should meet. A district plan is then arbitrarily generated to meet these districting objectives.

While systematically varying the control parameters, the simulation repeats these steps to create a selection of different samples. By comparing the properties of districts drawn using different parameter values we can analyze the effects of these parameters.

In fact, this procedure can be applied to a variety of formal districting rules. Because compactness standards are central to the current debate over redistricting, I use this procedure to focus upon plans drawn under equal population and compactness rules.[11]

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[11]You should note that, in these simulations, I model explictly only compactness and equal population rules. In particular,I do not require plans to be contiguous. I do this for several reasons:

• First, in order to isolate the effects of compactness, the constraint set and value functions were kept as simple as possible.

• Second, contiguity is often, in practice, an ill-defined or vacuous requirement. Practically any set of regions can be made contiguous if lines are drawn finely enough.

• Third, compactness requirements encompass contiguity: the maximally compact plan will not be measurably and avoidably noncontiguous.

The remainder of this section examines each step of the simulation in detail. First, I give a mathematical characterization of redistricting. Second, I discuss how we can measure the effectiveness of a redistricting rules through random sampling of district plans created automatically. Third, I describe the details of the simulation: how I create compact districts, how the I measure compactness, and how I model the geographical distribution of political groups.

3.3.1. Mathematically Characterizing Redistricting.

If you were a mathematically-inclined district planner you might characterize redistricting as a partitioning problem:[12] You would simplify the problem a bit by pretending that the state that you wish to redistrict is composed of indivisible census blocs.[13] Then, you would write out a function to evaluate partitions of blocs. Finally, you would solve for the maxima of the problem — you would find the partition with the highest value. If you could perform this procedure then you would have best possible district plan.[14] Adding compactness standards, does not make this problem more difficult to formulate. You can bring compactness standards change can be brought into the problem either as constraints to optimization, or as supplements to your value function.

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[12]See, for example, Gudgin and Taylor 1979.

[13]This is not far from the truth since most population data is, at best, limited to the census-bloc level of detail.

[14]A partition divides a set into component groups which are exhaustive and exclusive. More formally:

For any set $\mathbf{x} = \left\{x_1, x_2, ..., x_n\right\}$, a *partition* is defined as
a set of sets $\mathbf{Y} = \{\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_k\}$ $s.t.$

(1) $x_i \quad \mathbf{x}, \quad \mathbf{y}_j \quad \mathbf{Y}, s.t. \; x_i \quad \mathbf{y}_j$

(2) $i, \; j \neq i, \mathbf{y}_j \cap y_i =$

This characterization of redistricting is useful whether you are trying to find the least biased plan or the most effective gerrymander: If you are an altruistic social planner, you would use a value function that weighed all of the social benefits and costs of redistricting. An altruist might include such factors as preserving county boundaries, maintaining the competitiveness of districts, and minimizing the "bias" of the plan in your value function. (Lijphart 1989) A partisan, on the other hand, might attempt to maximize the number of safe party seats, or the probability of their party being in control of the legislature. Alternatively, a self-interested incumbent might attempt simply to maximize the probability of retaining her seat in upcoming elections.

Because altruistic social planners are likely to be outnumbered by partisans, incumbents, and other self-interested individuals, we may wish to impose rules on the redistricting process. Again, we can characterize these rules mathematically, as constraints on the set of feasible partitions. Whether we require that all districts have the same population, that they respect political boundaries, maintain geographically contiguity, or comply with compactness criteria, we can represent these requirements as constraints on our optimization problem. Approaching redistricting mathematically has two advantages: This approach can help us to draw better districts, and it can help us to predict the effects of particular redistricting rules.

3.3.2. Analyzing the Effects of Redistricting Rules.

How can we use a mathematical characterization of redistricting to predict the effects of redistricting rules? To answer this question turn to a concrete example: Suppose you are a party leader, intent on producing a partisan gerrymander, and suppose that a citizens' group introduces an initiative requiring all districts to be compact. Should you expend political resources to fight this initiative?

To make this decision you will have to estimate the effectiveness of the partisan gerrymander you expect to obtain when there are no rules, and then you must weigh that estimate against the effectiveness of the gerrymander which you expect to obtain if you are forced to draw contiguous districts. In mathematical terms, you subtract the value[15] of the optimal partition of the constrained problem from that of the optimal partition of the unconstrained problem. If the difference is big enough, then you should fight the contiguity rule.

In this example we assumed that the best plan that is found would then be *chosen*.. This corresponds to the situation in which an organized group has substantial control over the districting process.[16] In practice, even if we know which plan would make the best partisan gerrymander, we may not always be able to choose it. The plan we use may not be the result of a choice at all, but the result of a strategic game with multiple players.

For example, suppose that one party controls the redistricting process, but party leadership is weak and cannot force a particular choice of plans; instead, partisan incumbents compete to shape the redistricting plan to their liking. The plan that emerges from such a dynamic is neither “chosen” in a meaningful sense, nor is it necessarily optimal with respect to any participants values.

To predict the electoral effects of a redistricting rule upon this game, we would not examine the value of the optimal partition, but the value of the equilibria of the game under that rule. The redistricting rule may affect both values differently: A redistricting rule that makes it more difficult for an *individual* to create a gerrymander, might at the same time lower the stakes

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[15]Under uncertainty you might use the expected value of the plan instead.

[16]In essence, the “game” is played between a party and the courts. The controlling party submits a redistricting plan, and the court may then review it, and choose to modify, reject or accept it.

in the redistricting game enough to enable a *group* to cooperate upon a partican gerrymander. Such a rule would reduce gerrymandering when a single individual controls the redistricting process, but increase gerrymandering when a party controls the process. For example, when a state adopts term limits, incumbents will have less motivation to protect themselves, and they will be likely to loosen their grip on the redistricting process. Party professionals can then take up the slack.

In this paper, I limit the analysis to the former, decision-theoretic model. Despite the possibility of game-theoretic complications, it is valuable to understand how district rules affect the optimal plan. In some cases, a plan will, in effect, be chosen by party leadership or by some other unified group. In other cases, when the redistricting process is strategic, we still must understand the payoffs to players under different rules, i.e., the expected value of the optimal partition, before we can analyze equilibria of the game.

3.3.3. Creating Arbitrary Redistricting Plans.

While we can easily *formulate* redistricting as a partition problem, this problem may be difficult to solve. I use a number of techniques to draw plans that are near-optimal solutions to the partition problem.

Political scientists have used a number of different methods to search for compact districts. At the same time, computer scientists have developed similar techniques to search for optimal partitions. We can put these techniques into two broad categories:

• *Exact* methods systematically examine *all* legal districts either explicitly or implicitly. Explicit enumeration (“brute force” search) methods literally evaluate every possible plan. More sophisticated methods such as implicit enumeration and “branch and bound” limit the set of districts that must be searched by excluding groups of solutions that are obviously bad. After you have completed any of these enumeration techniques, you can find the optimal districts merely by sorting the list of district scores. Several authors have used these exhaustive methods to examine very small redistricting problems. (Garfinkel and Nemhauser 1970; Gudgin and Taylor 1979; Papayanopoulos 1973; Shepherd and Jenkins 1970)[17]

• *Heuristic* procedures use a variety of methods to structure the search for high-valued redistricting plans. None of the heuristic algorithms guarantee convergence to the optimal district plan in a finite amount of time. At best, they are good guesses.

I use exhaustive methods where possible.[18] Unfortunately, as the number of population blocs increases, the number of potential grows so rapidly that no computer can evaluate all of them.

In general, given *n* population blocs and *r* districts, the number of plans that you can create is shown in the following equation:[19]

$$\begin{aligned} S(n,r) &= \frac{1}{r!}\sum_{i=0}^{r}(-1)^i \binom{r}{i}(r-i)^n \\ &= \frac{1}{r!}\sum_{i=0}^{r}(-1)^i \left(\frac{r!}{(r-i)!\,i!}\right)(r-i)^n \end{aligned}$$

[17]A close examination of these algorithms reveals that in order to make the programs finish in a reasonable amount of time the authors use "short-cuts": For example, Garfinkel and Nemhauser (1970) use an "exclusion distance" to narrow the search for plans while Shepherd and Jenkins (1970) examine only "amalgamations" . Both authors assume, without proof, that these restrictions exclude only suboptimal plans.

[18]Exhaustive partitions can be obtained by the use of a restricted growth function. A *restricted growth function* on *n* is a vector

$$\begin{aligned} &\mathbf{v} = \{v_1,..., v_n\} \\ &\quad v_1 = 1 \\ &\quad \textit{and} \;\; v_i \le \max\{v_1,..., v_{i-1}\} + 1 \end{aligned}$$

see sec 1.5 of Stanton and White 1986 for an overview.

[19]*S* is known as a "Stirling Number of the Second Kind". See Even 1973 for a good introduction.

If we assume that a district is composed of exactly *k* tracts[20] then we can considerably reduce the number of plans, as the following equation shows: $S(n,r) = \frac{n!}{\frac{n}{r}!\left((n/r)!\right)^{\frac{n}{r}}} = \frac{n!}{k!\left((n/r)!\right)^{k}}$.

When there are even a moderate number of population blocs to be dealt with, no feasible methods exist that are guaranteed to find the most compact equal-population plan.[21] Many researchers in computer science and related fields believe that this type of optimization problem is inherently difficult to solve precisely.[22] In fact, many redistricting problems belong to a class of problems that computer scientists have labeled "computationally intractable".[23] (Garey and Johnson 1983; Johnson 1982; Johnson 1984b)

Because of these barriers to finding an exact solution to larger problems, in the larger simulations I turn to heuristic methods for finding compact districts. Most heuristics for locating optimal partitions are based on the principle of iterative improvement. In the simplest of these methods, known as *hill climbing*, you start with a set of randomly generated redistricting plans and repeatedly look for small changes to the plan that improve it — stopping when there are no

[20]This may be a more practical approximation than the first equation because districts are supposed to be equal in size, and because census blocs are often close in size.

[21]For example, here are 5.8 * 10^{31} possible district plans when n=50, k=25, and r=2, far too large to examine exhaustively .

[22]For some restricted cases, notably where only two districts are being created, the problem can be made tractable. (Hershberger 1991)

[23]Technically, these problems belong to the set of problems that computer scientists have labelled "NP-Complete". Most computer scientists believe that to solve an NP-complete problem requires you to spend computation time that grows exponentially with the problem's size ("n" above). Since the computational requirements for these problems grow so quickly, you will be only be able to solve the smallest instances of them. For some NP-Complete problems, however, you can quickly find appoximate solutions, or you can impose restrictions on the demain of the problem that make it easier to solve. This barely scratches the surface of the issue, but a further discussion of NP-completeness is beyond the scope of this paper — see Papadimitriou 1994 for an introduction.

small alteration can yield an improvement.[24] Several previous researchers have used variants of hill climbing methods to draw new district plans or to make improvements to existing districts (Liittschwager 1973; Moshman and Kokiko 1973; Nagel 1972; Rose Institute of State and Local Government 1980; Vickrey 1961; Weaver and Hess 1963) In the illustration below, I show an example of this process:

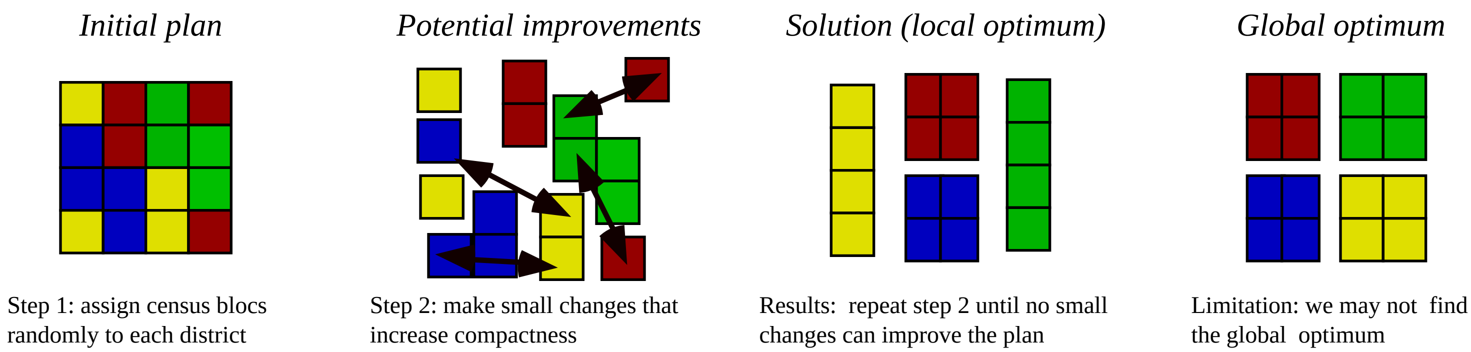

Figure 3: an example of creating a compact plan through "hill climbing"

Simple hill climbing methods, however, sometimes produce results that are far from optimum, because these methods are easily trapped in local optima. To minimize this problem in the simulations, I supplement hill-climbing methods with a number of methods that have been successfully used to solve similar problems in other fields: simulated annealing,[25] genetic

[24]•I use a variant of hill-climbing similar to Nagel's method. I start with a plan drawn at random. I then examine all possible single-unit exchanges; if the plan can be made more compact by moving a census bloc from one district to another, then the trade is made. If more than one good exchange is possible, the exchange is made that results in the most improvement. I repeat this examination and trading process until no more good trades are left.

In some limited cases, it is possible to derive the most compact plan from geometric arguments. In these cases, in addition to the other methods I use a variation of hill-climbing that I call "descent": I start with the most compact plan and use a simple hill-climbing to create less compact variants of it.

[25]•*Simulated annealing* is one of the most successful of combinatorial optimization methods. It is based on a mathematical analogy to the slow cooling of metal. If the value function being optimized is sufficiently well behaved, simulated annealing asymptotically converges to the optimum value. (van Laarhoven and Aarts 1989) The algorithm has been used for other partitioning problems in integrated circuit design (Zissimopoulos, Paschos and Pekergin 1991), and has been previously recommended for use in redistricting. (Browdy 1990b)

I use a variant of the annealing algorithm known as *adaptive simulated annealing*. This variant is often more successful than standard annealing because it adjusts itself to the particular behavior of the value function.[25] This algorithms,[26] and monte-carlo methods. In addition to producing better district plans, by using a variety of different methods I ensure that my results are not being driven by quirks in the optimization process — my results reflect properties inherent in compact districts.

I cover the range of procedures that are available for drawing compact districts. Should compactness standards be legally mandated, district planners will have little choice but to turn to such techniques. Thus the plans that I produce, though simpler than real district plans, are similar in principle to those that will be produced should compactness standards become widespread.

3.3.4. Measuring Compactness.

All of the optimization methods that I have discussed are flexible enough to accommodate a variety of value functions. Yet, choosing a particular compactness measure was a special challenge because there are so many different measures available. In fact, previous researchers have proposed over thirty distinct measures of compactness, a plethora that makes selecting a single method difficult. (Altman 1995; Niemi, et al. 1991; Young 1988) Neither the courts nor political scientists recognize a single standard for measuring compactness, and

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variant of annealing has been found to be very successful in comparison with other optimization algorithms in such diverse fields as electrical engineering, biology and finance.(Ingber 1993)

• *Monte Carlo* methods generate large numbers of potential solutions randomly, and select the solution with the highest value. I use these techniques to augment hill-climbing, by providing a variety of different starting points from which we can make improvements. The particular Monte Carlo procedure that I use is as follows: I create a district by adding population units at random, until uthe districts are approximately the correct size.

[26] *Genetic algorithms* are search algorithms based on an analogy to natural selection and genetic combination. Potential solutions to the optimization problem are defined as genetic strings, which can be mutated or "crossed" with other strings. A group of potential solutions then competes to survive and reproduce in the next generation. (Chandrasekharam, Subhramanian and Chaudhury 1993) demonstrates the effectiveness of genetic algorithms for graph-partitioning problems, which are somewhat similar to the redistricting problem.

I use a variant of the genetic algorithm employing mutation, partially-matched crossover, and inversion. See (Goldberg 1989) for a detailed explanation of these variations.

although many states require compactness only three states (Iowa, Colorado, and Michigan) define the term.(Grofman 1985) What compactness measures are most representative of the set?

Most measures in the compactness literature fall into one of three categories: "area-based" measures, "perimeter-based" measures, and "population-based" measures. For the simulations,I chose a compactness standard from each of these categories. In addition, each of the measures that I use duplicates, as closely as is practical, a measure currently in force in the United States.[27]

I measured compactness in the following ways:

• To measure the compactness of a district's area, I compared the area of the district to the area of the smallest box bounding that district.[28] A plan's compactness is defined as the mean compactness of its districts. This measure duplicates, as much as possible, a compactness requirement used in Iowa and in Michigan.[29]

• To measure the compactness of a plan's boundaries I calculated the total perimeter of all it's districts; the best plan minimizes this total.[30] This measure is currently used by Colorado to evaluate districts. (Grofman 1985)

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[27]Each of these measures is required by a state statute or constitution, but compactness requirements seem often to ignored in practice.

[28]Some similar compactness measures use a bounding circle or convex polygon. While these other shapes are theoretically preferable (Altman 1995) I use a bounding box here because of the discreteness of the simulation map, and the desire to have my standard duplicate current legal standards. Furthermore, given the large granularity of population blocs in this simulation this measurement is quite similar to comparing districts to the bounding circle, while being more efficient to compute.

[29]The 1980 Iowa General assembly Bill generally defines compactness as "Compact districts are those which are square, rectangular or hexagonal in shape to the extent permitted by natural or political boundaries." The Michigan constitution also specifies, generally, that its state house districts should be "as nearly square in shape as possible". Iowa also offers several operational definitions of compactness, the first of which is "the absolute value of the difference between length and width". (From (Grofman 1985). 180 fn) While this absolute value is not equivalent to my bounding-box measure, it is a similar, if cruder, attempt to capture the squareness of a district. (Altman 1995)

[30]But note that compactness scores using this method do not fall on the (0,1] interval.

• To measure the compactness of a district's population,[31] I calculated the population moment of inertia for the district.[32] A plan's compactness is defined as the mean compactness of its districts. This approach is similar to the measure in force in Iowa, the only population measure currently in effect in the U.S.[33]

For all of these measures, where possible I normalized their scores to fall within the (0,1] interval. Plans that have a score of "1" are as compact as possible.[34]

3.3.5. Simulated Politics.

To simulate the of compactness on elections we must make some assumptions about the behavior and distribution of political groups. I assume that each group runs a candidate in each district, and members of that group will vote "sincerely" for that candidate: Democrats always vote for the available democratic candidate, and do not become Republicans merely because they are moved to a different district. This assumption is best fitted to polarized, bi-partisan elections; where there is a viable third candidate, voters may have an incentive to vote strategically, and where there is less polarization between political parties the effect of particular lines will be less.

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[31]Since in the simulations, all population blocs have the same weight (although the partisan proportion in each bloc may vary), population-based measures produces results identical to analogous area-based measures.

[32]Formally, this is $\frac{P}{\sqrt{\sum_{x \in X} \left(p_x d(x)\right)^2}}$, where P is the total population of the district, p_X is the population of a particular census bloc, X is the set of all census blocs in the district, and *d()* is the geographical distance from the center of the census bloc to the population-center of the district.

[33]The Iowa measure calls for taking the ratio of the "dispersion of population" around the population center of the district, to the dispersion around the geographic center of the district. Since in this simulation I use only maps with uniform population densities, these two centers are always identical, and hence this ratio will always be one. Under these conditions, the population moment of inertia measure that I use better captures population dispersion.

[34]This normalization was not always possible for the perimeter measure, since it is not always possibible to know the value of the perimeter of the most compact plan. Where I give raw perimeter scores, remember that plans with a greater perimeter are *less* compact.

I assume that while different political groups have the same voting habits, they may be distributed across the state "map" differently. In the simulation, I maps are represented by *mxn* grids. Each map is inhabited by two different political groups. To isolate the effect of compactness from the effects of equal population standards, each census bloc is normalized to have one hundred voters, so that only the proportion of each type of voter varies across population blocs. I duplicated the simulations using three different models for to determine the political composition of each census bloc.

I first performed a set of simulations using a very simple population model that I will refer to as the "uniformly-random" distribution. In this model I assume that each census bloc has roughly the same number of members from each political group, subject to random variations. In particular, that the population of each census bloc is drawn from the same normal distribution.[35]

This simple model does not produce much population clustering, which is unfortunate because clustering is a feature of most population geography models. (Garner 1969) In the second set of simulation runs, I used a "clustered" distribution to model the distribution of political groups. In this "clustered" distribution one political group is concentrated into *r* compact clusters, each of size *s*, and each randomly located. Similar cluster models have been used previously to explain voting behavior; in particular, Gudgin & Taylor (1979) find that the well known "cube law" [36] can be explained by a variation of the cluster population model.

[35]I duplicated each of the simulations that I this model, substituting uniform distributions (with the same mean) for normal distributions, but the simulation results were indistinguishable.

[36]The "cube law" is the hypothesis that as voters "swing" from one party to another, the number of seats actually won or lost in the swing varies as the cube of the size swing. (In other words, it hypothesizes a certain form for the elasticity of seats w.r.t. change in votes.)

In the third set of simulation runs, I use a more complicated clustering process that is based on Schelling's (1978) neighborhood formation model. In Schelling's model, persons in two different groups are at first randomly distributed on a map, then if an individual is surrounded by too many individuals of the other type they can move to any adjacent square, if they prefer. In each round, every person is offered an opportunity to move; when no one can improve their location by moving, the neighborhood is 'stable'. (Schelling 1978) My model is similar to Schelling's, although it is aggregated at the census bloc level.[37] I show a typical population distribution that was generated by this model in Figure 4:

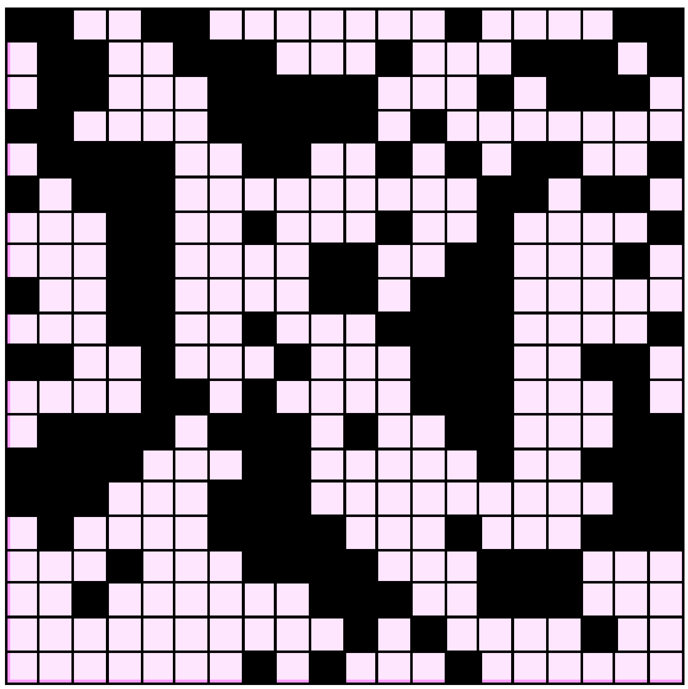

Figure 4: A map of 20x20 census blocs, with two political groups distributed across it using a Schelling distribution. The black squares represent census blocs primarily occupied by the minority political group.

[37]Schelling requires that individuals move into empty spaces, whereas I allow two willing individuals to trade places.

4. Evaluating the Electoral effects of Compactness : Simulation Results

Before I discuss the results, let me briefly remind you of the steps in the simulation process:

• First, I decided on a "map" size. I duplicated the simulation runs with rectangular maps ranging in size from a 2 by 4 map with 8 census blocs to a 20 by 20 map which contained 400 census blocs.

• Second, using the uniformly-random, clustering, and Schelling models, I distributed two political groups across the map.

• Third, I used the combinatoric-optimization algorithms to create arbitrary and equal-population districting plans which had a range of compactness values.[38]

• Fourth, I simulated an election for each plan and distribution.[39]

• Finally, I analyzed the relationshionship between the compactness of a districting plan, and the success of a particular political group.

The results from these thousands of simulation runs reveal three interesting properties of compact plans and of compactness standards. First, the distribution of compact plans shows that compactness measures are useful only for comparing similar plans, but not for making absolute measurements of plans. Second, the simulation shows the difficulty of drawing compact plans under some measures — we should avoid these particular measures if we want to minimize the

[38]Since only the hill-climbing and descent algorithms had been successful at producing compact plans, only these algorithms were used to generate "sample" plans. The other algorithms had to be discarded for this purpose because they produced extremely noncompact plans only — resulting in a sample with very small variance in compactness, or because they failed to converge to generate a solution in a reasonable time. For this same reason, only the perimeter measure of compactness was used.

[39]I examined a total of 10,000 plan/population distribution combinations were examined for each simulation run.

potential for gerrymandering. Finally, the simulation shows that district compactness can systematically influence election results.

4.1 Compactness Measures are Relative Measures

I used an exhaustive analysis of districts to generate all possible plans for a number of small maps. In Figure five I use a box-plot[40] to compare the distribution of compact plans under the perimeter measure, for different map sizes, shapes, and numbers of districts[41]:

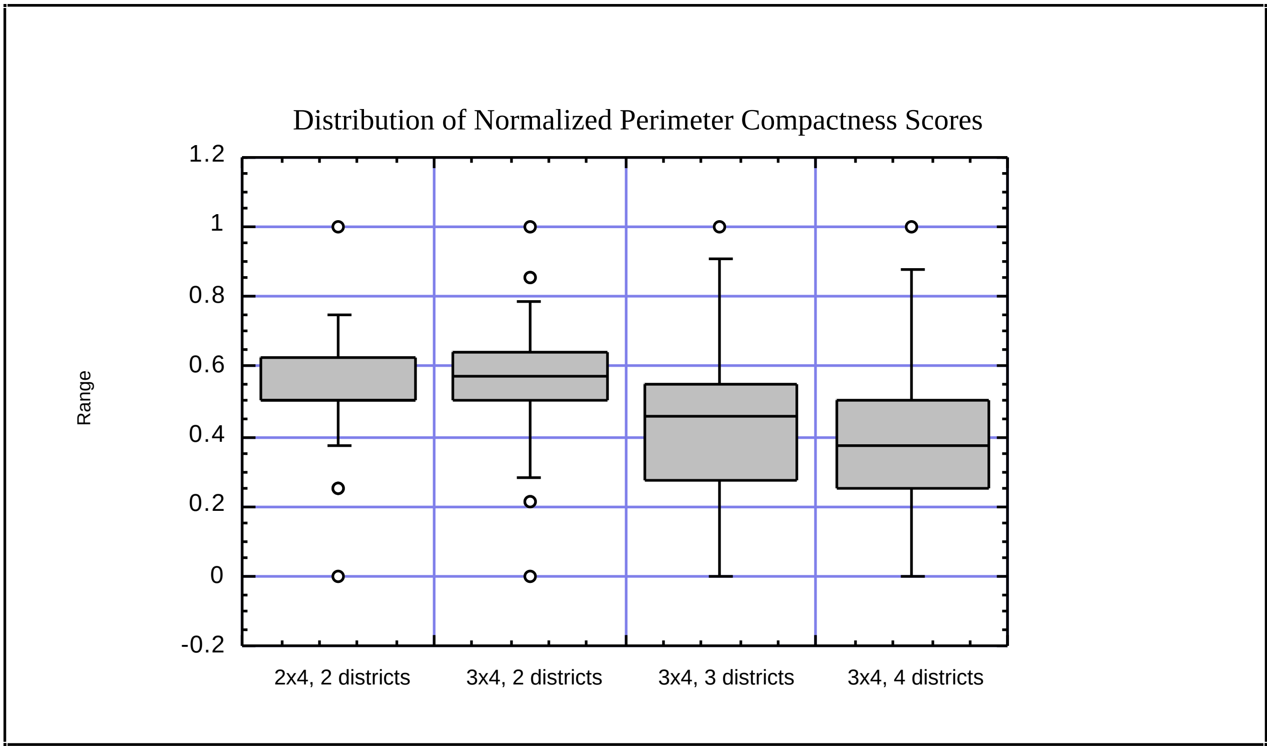

Figure 5 Box plots of Plan Compactness. Perimeter measure is normalized to (0,1)

[40]Box plots are commonly used to compare distributions. In these plots, the top and bottom of the box correspond to the 25th and 75th per- centiles of the variable, while the whisker lines extend beyond the box by 1.5 times the interquartile range (so that approximately 99 percent of normally distributed data will lie within them.) The median is identified by a horizontal line, and outliers are identified by the small circles.

[41]See (Altman 1995) for a detailed description of the distributions of a wide variety of compactness scores.

Looking at any one of the box-plots in Figure five, you can easily see that compact plans are scarce — most plans fall *far* short of relative to the optimum. Furthermore, the scores of most plans cluster in a very narrow range of compactness values.

If you compare the box-plots for different maps, you can see that while the distribution of scores for each map is similar, the values of the minimum, maximum, and median scores are quite different. Given only the compactness scores of two plans, you can make reasonable comparisons between them only if these plans partition the same map into identical numbers of districts — a compactness score is meaningless outside of its specific context. For example, a score of ".5" is in the bottom decile of plans for the first map in Figure five, and in the *top* decile for the last map.

Some authors have proposed that a minimal level of compactness be mandated for district plans. Given the shape of the distribution of compactness scores observed here, the effectiveness of compactness standards for limiting manipulation is likely to be very sensitive to the particular minimum level specified. If the minimum level is set high, the vast majority of plans will fail to meet the standard — it may be difficult to draw any plans at all. If, on the other hand, it is set at the middle of the distribution, the ability to gerrymander may be virtually unaffected.

Empirical studies of compactness scores must also take note of both their nonlinearity and their sensitivity to geographic context. Suppose that your ability to gerrymander is roughly proportional to the number of plans from which you can choose: you will find it immensely more difficult to create an effective gerrymander that scores in the top 99th-percentile than to draw a plan with a slightly lower relative score. Furthermore, since compactness scores will depend on state boundaries, you may find it easy to create a gerrymander that scores ".90" in a state with regular boundaries, like Iowa, and impossible to create any plan at all that scores above ".75" in a state like Maryland. In general, comparing the compactness of plans across different states[42] has little value.

4.2 Some Compactness Standards Make Detection of Gerrymanders Difficult

In Table 1, I compare the performance of each optimization method.[43] Both the hill-climbing method and the genetic algorithm were equally successful in finding optimal plans, although the genetic algorithm was too computation-intensive to be used on the larger maps.[44] Unlike these two methods both the monte-carlo procedure and simulated annealing performed poorly.[45] I must caution that these results are preliminary — the relative performance of these procedures could change in a more complex districting environment, with a different value function, or with a different variant of each of these methods.

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[42]Differences in population distribution can be further expected to cloud such comparisons, as the ability to draw compact districting plans will be affected by equal population constraints.

[43]This measure is limited to cases where the optimal plan can be deduced from regularities in the shape and population distribution.

[44]The time required to find a solution using the hill climbing method seemed to grow quadratically in the number of census blocs (in time-complexity notation: $O\left(k^2\right)$) while the convergence rates for genetic algorithms and simulated annealing grew at an even faster rate. Consequently, for maps larger than 5x5, I used descent and hill-climbing methods exclusively.

[45]Given the bottom-heavy distribution of scores, which I showed in figure 5, I expected the monte-carlo procedure to perform poorly. I was initially puzzled, however, by the poor performance of simulated annealings, a performance not at all in keeping with its track record. In retrospect, however, this puzzle is explicable: The annealing procedure that I used sometimes made trades that would cause the population of the districts to become unbalanced, lowering the overall score of the plans. Once it did this, it was usually unable to recover because future changes to the plan were unlikely to bring the plan back into balance. In the future, I will probably be able to improve the performance of annealing by combining it with other special-purpose methods that take into account peculiarities of the optimization environment.

Grid Size	Number of Districts	Number of Possible Plans	Compactness Measure	Best Possible Score	Mean of 1000 Random District (std. dev.)	Hill-Climb mean (std. dev)	Anneal mean [46] (std. dev)	Genetic mean (std. dev)
3x4	4	15400	perimeter	32	42 (3.0)	32.1 (0.41)	-------------	32 (0)
3x4	4	15400	area	0.75	0.33 (0.07)	0.73 (0.07)	0.45 (0.096)	0.73(0.03)
3x4	4	15400	moment[47]	1.5	0.98 (.12)	1.0 (0.02)	1.2 (0.1)	1.49 (0.06)
5x5	5	5.2×10^{12}	perimeter	unknown	86 (4.6)	54.8 (2.8)	-------------	55.5 (4.3)
5x5	5	5.2×10^{12}	area	unknown	0.23(.03)	0.42 (0.065)	0.02 (0.07)	0.42(0.046)
5x5	5	5.2×10^{12}	moment	unknown	0.61 (0.04)	0.99 (0.001)	-------------	1.03 (0.05)[48]
8x8	4	$5.0\text{x}10^{53}$	perimeter	64	203 (9.3)	90.5 (8.7)	-------------	-------------
8x8	4	$5.0\text{x}10^{53}$	area	1.0	0.25 (.006)	0.27 (0.015)	-------------	-------------
8x8	4	$5.0\text{x}10^{53}$	moment	1.0	0.34 (.007)	0.65 (0.02)	-------------	-------------
9x9	9	$1.5\text{x}10^{65}$	perimeter	108	295 (7.3)	150 (9.8)	-------------	-------------
9x9	9	$1.5\text{x}10^{65}$	area	1.0	0.12 (.006)	0.18 (0.026)	-------------	-------------
9x9	9	$1.5\text{x}10^{65}$	moment	1.0	0.32 (.009)	0.87 (0.03)	-------------	-------------
20x20	8	$3.0\text{x}10^{728}$	perimeter	160	1508 (13)	-------------	-------------	-------------
20x20	8	$3.0\text{x}10^{728}$	area	1.0	0.06 (.001)	-------------	-------------	-------------
20x20	8	$3.0\text{x}10^{728}$	moment	1.0	0.13 (0.0)	0.36 (0.003)[49]	-------------	-------------

Table 1: Performance of Algorithms using Different Measure of Compactness[50]

In fact, the most striking differences in this chart are not among methods, but among compactness measures. In Figure 6 I show the ratio ("approximation ration") between the mean value of the plans created using the best optimization method to the value of the most compact

[46]Annealing returned plans that violated equal population constraints, these plans were assigned a compactness value of 0.

[47]If measured over a continuous area, the moment of inertia measure for a shape can be no larger than one, but this condition is violated in very small discrete approximations.

[48]Sample size in this case was 148, because computation exceeded time limit.

[49]Sample size in this case was 287, because computation exceeded time limit.

[50]For each table entry I performed 1000 simulation samples, unless otherwise noted. When a cell filled is filled with dashes it means that the specified algorithm was not able to complete a significant number of iterations before the time limit (several days) expired.

plans possible. Notice that, in general, these methods were much more successful at finding compact plans under the perimeter standard and moment-of-inertia standard than under the area-based standard. What does this tell us about the properties of these different standards?

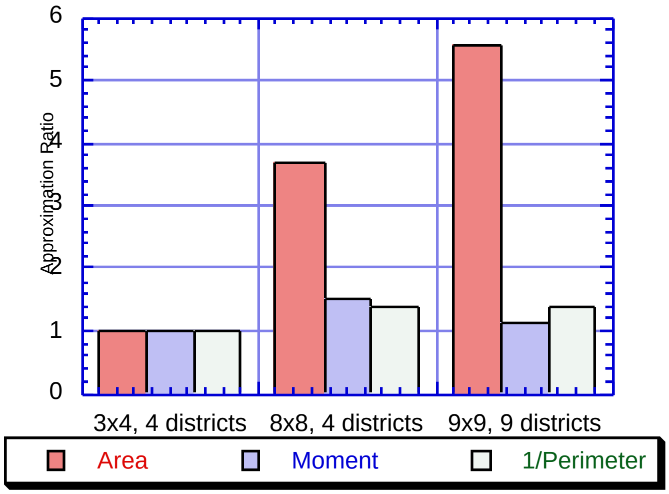

Figure 6: To obtain an approximation ratio, I divide the best possible score by the mean reached by the best algorithm.[51] The best possible ratio is one, which means that the algorithm always reaches the best possible solution.

Remember that all of these optimization methods are based upon iterative improvement, i.e., they operate through gradual limited change. Since these methods work well for the perimeter-based standard (as demonstrated by Figure 6), we can conclude that the perimeter and moment standards are sensitive to small changes in a district plan; whereas, the area-based measure is much less sensitive to small changes— to improve the plan's area compactness we need to change districts radically.

[51] I use an the inverse of the perimeter here because the perimeter measure grows when a shape becomes less compact, unlike the others.

Changing districts radically can be politically difficult and can also interfere with other redistricting goals such as preserving natural boundaries and communities of interest. While the simulations ignore these concerns, the courts should not. Because of these difficulties, the courts will find the perimeter-based standard easier to manage than the area-based standard.[52]

Furthermore, for most real district maps we will not know the value of the most compact plan beforehand, a situation that is exacerbated by the area-based measure. Since it is likely to mislead us with plans that are locally optimal, but which fall far short of the most compact plans, the area-based measure allows gerrymanderers much greater leeway in designing their districts. Altruistic district planners will suffer as well, as they may expend unnecessary effort trying to improve a plan that is already very close to optimal.

4.3 Compactness Standards May Create Opportunities for Political Manipulation

In addition to failing to prevent gerrymanders, there is a further consideration that has not been suggested in the literature as yet: The process of evaluating plans under a compactness standard might well induce strategic behavior that would harm the reapportionment process.

Finding the maximally compact plan is, as I have indicated, a very difficult mathematical problem. In practice, it will often not be possible to determine whether a plan is “optimally compact”, especially for plans composed of a large numbers of census blocs. And, as I argued in the section 4.2, the simulation results indicate that if we do not know the value of the optimal plan, we cannot set reasonable an “absolute” compactness limit.

[52] In coming to this conclusion, I retain the assumption (from section 3.3.2) that a single party substantially controls the redistricting process, and is able to create plans that the court must then either approve, modify, or reject.

Instead of using some absolute measuring value, all we will have to compare plans against will be each other, or else we can look for improvements that can be made to plans. It has even been suggested that when two plans are proposed, the most compact should automatically be implemented. (Polsby and Popper 1991)

Unfortunately, in a strategic political environment, where plans are compared only with each other, the very shape of districts becomes valuable information to your opponents: If you hide your plan, there is a chance that opponents will mistakenly believe their plan to be the most compact, which is to your advantage. Whereas, if you reveal your plan you give up this strategic advantage without gaining anything. In sum, because compactness standards give district planners an incentive to hide information these standards may increase political manipulation.

4.4 Compactness standards are not politically neutral

In this next section, I will show that there is a systematic relationship between compactness standards, population distribution and electoral advantage. The specific effect that these standards will have on redistricting, however, will depend on the political institutions used to create districts.

4.4.1 Arbitrarily Selected Compact Districts

Polsby and Popper 1991 claim that if the court adopt a policy of automatically accepting the most compact districting plan proposed to them, then through competition among political groups gerrymandering will disappear and we will end up with arbitrary and compact districting plans. This view is relatively recent, but scholars have long argued that we should simply use a computer to generate arbitrary district plans, following only the principles of compactness, contiguity and population equality.(Harris 1964; Kaiser 1966; Weaver and Hess 1963) Suppose that we did manage to create districts arbitrarily, following only the principles of compactness and population equality, as these scholars desire. Would this be a neutral solution to the gerrymandering problem?

In Table 2 I show the correlation[53] between compactness and electoral results in such a case. As I had suggested earlier, the electoral effects of compactness depend upon on the geographic distribution of political groups.

[53]Since neither compactness scores nor seats were distributed normally, I also report Somers's d along with the correlation measures. Somers's d is a nonparametric measure of association that is quite similar to the more well known Kendall's Tau. Somers's d differs from Kendall's Tau in that it treats ties assymetrically, ignoring ties on the dependent variable (number of minority seats, in this case). I use it in this case because of the number of minority districts takes on only a few values, leading to many ties that would distort Kendall's measure. See (Liebetrau 1983) for a discussion of these measures.

Grid Size	Number of Districts	Cluster Size	Number of Clusters	Minority Percentage of Population	Mean Minority Controlled Districts (std. dev.)	Correlation Between Minority Controlled Districts and Compactness (Somers's d)
5x5	5	1	3	12%	0.02 (0.13)	0.0 (-0.03)
5x5	5	1	5	25%	0.34 (0.49)	0.0 (-0.01)
5x5	5	1	12	48%	2.37 (0.6)	0.06 (0.00)
5x5	5	4	1	16%	0.35 (0.47)	0.39 (0.44)
5x5	5	4	2	32%	1.17 (0.63)	0.36 (0.35)
5x5	5	4	3	48%	2.37 (0.6)	0.07 (0.08)
5x5	5	9	1	36%	1.46 (0.6)	0.32 (0.26)
8x8	4	1	26	40%	0.47 (0.43)	0.01 (-0.02)
8x8	4	9	3	42%	0.93(0.65)	0.46 (0.47)
8x8	4	16	1	25%	0.24(0.42)	0.46 (0.61)
8x10	8	9	3	34%	1.38(0.89)	0.58 (0.52)
20x20	8	1	100	25%	0.04(0.20)	0.0 (0.0)
20x20	8	4	40	40%	3.1 (1.2)	0.29 (0.23)
20x20	8	9	18	41%	3.84(1.85)	0.43 (0.33)
20x20	8	36	2	18%	1.38(1.03)	0.66 (0.65)
20x20	8	36	4	36%	4.1 (1.56)	0.58 (0.52)

Table 2: Perimeter Based Compactness Effects on Minority Representation with Clustering Distributions. [54](10,000 Samples were performed for each grid/district combination)

When all political groups are thoroughly geographically mixed, no district, compact or not, can contain a majority of a minority group. Even when political groups form small geographic clusters, if these clusters are geographically dispersed, as in the uniform and normal population distribution models, then compact districts are no more likely to elect candidates from one political group than from another. This is not because the district drawing process is politically neutral in these circumstances. On the contrary, in these cases compactness has no

[54] I generated ten thousand district plan-population combinations for each set of district and population parameters. Since I controlled for the other parameters by keeping them constant across runs, a standard correlation measure adequately represents the linear association between the number of districts captured by a minority and the compactness of districting plans. Genetic algorithms, hill climbing and descent methods were used for the 5 by 5 case, while hill-climbing and descent methods were used for all other cases. Since the results from each method were similar, only hill-climbing results are reported.

effect on electoral outcomes because geographical districting itself embodies such a powerful majoritarian bias that minority political groups are unlikely to win seats under *any* circumstances.[55]

Consider the 12th entry in the table, which describes a simulation run on a 20 by 20 grid where the minority political group populated 100 of four-hundred census blocs in the state. Although the minority group makes up 25 percent of the voting population of the state, it loses nearly every election simply because it has the misfortune not to be geographically concentrated. The only way for minority political groups to win any seats in these circumstances would be if we tailored districts expressly to their boundaries, linking small concentrated clusters or minorities. While districts created in this way will almost certainly be noncompact, for such purposes *non*compactness will be a necessary condition, but almost never a sufficient one. Compactness makes it impossible for dispersed minority political groups to gain representation. I will return to this issue in section 4.4.2.

These results may understate the electoral effects of compactness on minority representation when we consider the assumptions we made in the simulation about turnout. In the simulations we assumed that each political group turned out to vote at the same rate and voted strictly for their own party, a simplification that helped to reveal the general dynamics of voting in compact districts, but which may bias our predictions. In particular, if minorities that are geographically dispersed also turn out at a lower rate than the majority political group, or have a

[55]I use the term “bias” here in the descriptive, rather than normative sense — arbitrarily drawn districts tend to award district-share in excess of the majority’s share of the population. (See Grofman for a similar analysis of the majoritarian bias inherent in redistricting. (Grofman 1982)) A majority bias, as defined in this way, is not necessarily contrary to the public interest. At the same time, I disagree with Engstrom and Wilder (1977) when they argue that randomly-drawn districts should be used as the yardstrick of procedural fairness; random districts so predictably favor an identifiable group that they can hardly be said to be a level playing field.

higher rate of cross-over voting, it will be even more difficult to draw districts compact districts which allow minorities an opportunity to elect a candidate of choice.

In contrast, compactness helps combat majority bias when minority political groups are geographically concentrated: as Figure 7 shows, there is a strong positive correlation between a plan's compactness and the number of seats captured by such a political group. Why do we see such a correlation? The explanation for this is straightforward: When both districts and minorities are very compact, a concentrated minority will sometimes, fortuitously, fall completely within the district lines of the "optimal" plan. Under any other circumstances short of a purposeful minority gerrymander, by contrast, the majoritarian bias inherent in geographical redistricting makes minority controlled districts extremely unlikely.

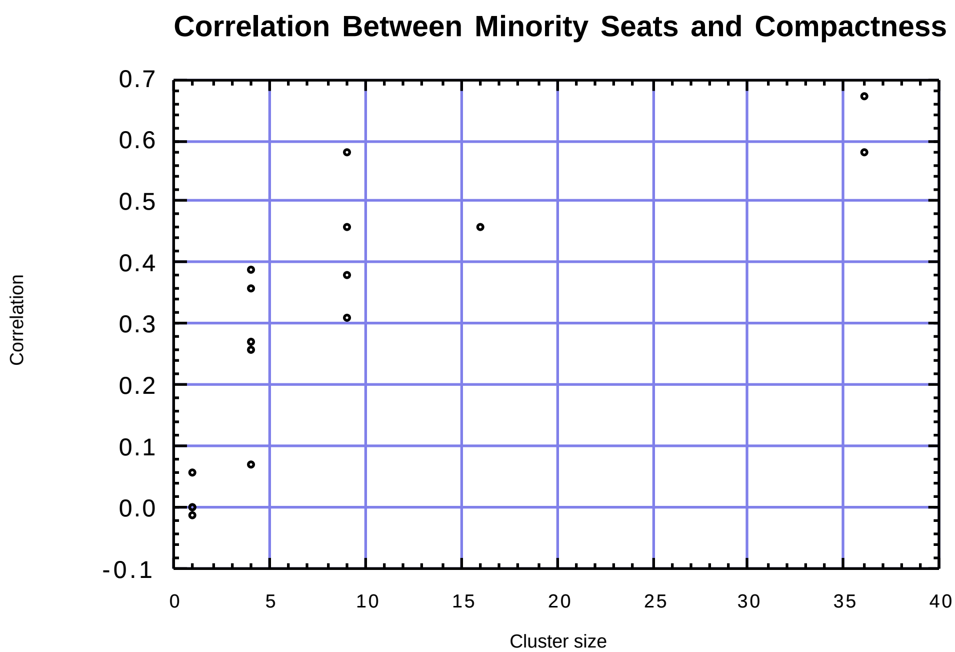

Figure 7: Correlation between minority seats and perimeter compactness (based on data in Table 2)

This phenomena is not isolated to perimeter compactness, to small numbers of districtits, or to compact clusters. Table 3 shows us the same patterns when we use the moment-of-inertia measure for compactness. Table 4 shows us somewhat weaker patterns when minorities are grouped in less compact clusters and into more districts.

Grid Size	Number of Districts	Cluster Size	Number of Clusters	Minority Percentage of Population	Mean Minority Controlled Districts (std. dev.)	Correlation Between Minority Controlled Districts and Compactness (Somers's d)
8x8	4	1	26	40%	0.47 (0.53)	-0.01 (-0.01)
8x8	4	4	6	38%	0.48 (0.55)	0.14 (0.15)
8x8	4	9	3	42%	1.2 (0.73)	0.52 (0.52)
8x8	4	16	1	25%	0.41 (0.49)	0.53 (0.57)
8x10	8	9	3	34%	1.38 (0.93)	0.62 (0.55)
20x20	8	4	40	40%	0.76 (0.72)	0.15 (0.12)
20x20	8	9	18	41%	1.46 (0.86)	0.25 (0.19)

Table 3: Moment-Of-Inertia Based Compactness Effects on Minority Representation with Clustering Distributions. (10,000 Samples were performed for each grid/district combination)[56]

[56]Hill-climbing and descent methods were used for these cases. Since the results from each method were similar, only hill-climbing results are reported.

	Grid Size	*Number of Districts*	*Minority Percentage of Population*	*Mean Minority Controlled Districts (std. dev.)*	Correlation Between Minority Controlled Districts and Compactness (Somers's d)
Perimeter	8x10	4	34	0.33 (0.48)	0.10 (0.12)
	20x20	16	10	0 (0)	0 (0)
	20x20	16	25	0.08 (0.28)	0.08 (0.16)
	20x20	16	40	2.88 (1.14)	0.25 (0.19)
Moment	8x10	4	34	0.37 (0.49)	0.01 (0)
	20x20	16	10	0 (0)	0 (0)
	20x20	16	25	0.08 (0.28)	0.04 (0.06)
	20x20	16	40	2.87 (1.11)	0.13 (0.09)

Table 4: Compactness Effects on Minority Representation with Schelling Distributions.
(10,000 Samples were performed for each grid/district combination)[57]

4.4.2 Gerrymandering Compactly

The previous section examines the effects of compactness when the creation of district plans is in some sense arbitrary. What are the effects of compactness when one party purposefully creates a districting plan?

Obviously, if one party is in complete control of drawing districts all of the time and in all places, its ability to create districts will be limited by practically any restriction, compactness included. Suppose however, that different parties substantially control the redistricting process at different times and different places. If the courts continue to use compactness as a red flag to mark plans for judicial review, the party in control may try to produce the most compact partisan gerrymander that they can. How will compactness affect electoral results in these circumstances?

[57] Hill-climbing and descent methods were used for these cases. Since the results from each method were similar, only hill-climbing results are reported.

Tables 5 and 6 compare the relationship between compactness and seats for the minority party when compact districts are arbitrarily selected to the case where partisans try to gerrymander compactly. In this second case, I altered the simulations to find, for each party, the most compact plan subject to the constraint of partisan seat maximization.[58]

	Grid Size	*Number of Districts*	*Cluster Size*	*Number of Clusters*	*Minority Percentage of Population*	*ARBITRARY PLANS Correlation Between Compactness and Minority Controlled Districts (Somers's d)*	*COMPACT GERRY -MANDER Correlation Between Compactness and Minority Controlled Districts (Somers's d)*
Perimeter	8x8	4	1	26	40%	0.01 (-0.02)	-0.20 (-0.19)
	8x8	4	9	3	42%	0.46 (0.47)	0.23 (0.22)
	8x8	4	16	1	25%	0.46 (0.61)	0.18 (0.22)
	8x10	8	9	3	34%	0.58 (0.52)	0.19 (0.19)
	20x20	8	1	100	25%	0.0 (0.0)	0.00 (0.00)
	20x20	8	4	40	40%	0.29 (0.23)	-0.23 (-0.22)
	20x20	8	9	18	41%	0.43 (0.33)	-0.02 (-0.02)
	20x20	8	36	2	18%	0.66 (0.65)	-0.15 (-0.27)
	20x20	8	36	4	36%	0.58 (0.52)	0.06 (0.06)
Moment of Inertia	8x8	4	1	26	40%	-0.01 (-0.01)	-0.58 (-0.65)
	8x8	4	4	6	38%	0.14 (0.15)	-0.45 (-0.44)
	8x8	4	9	3	42%	0.52 (0.52)	0.14 (0.16)
	8x8	4	16	1	25%	0.53 (0.57)	0.09 (0.12)
	8x10	8	9	3	34%	0.62 (0.55)	0.27 (0.24)
	20x20	8	4	40	40%	0.15 (0.12)	-0.63 (-0.64)
	20x20	8	9	18	41%	0.25 (0.19)	-0.56 (-0.55)

Table 5: Compactness Effects on Minority Representation with Clustering Distributions for Compact Gerrymanders [59](500 samples were for each grid/district combination)

[58]Gerrymandering to capture the maximum number of seats is only one possible partisan objective. In the real world, where there is uncertainty over voters behavior, partisans might try to maximize the probability of controlling the legislature instead. I use seat maximization in this simulation because I have assumed certainty, and because for some minority population distributions, it may be impossible a-priori to capture the legislature.

For a number of technical reasons, this turns out to be a much more difficult problem computationally, which is why I was able to run only 500 samples for these cases. Even these 500 samples required substantial processing time on a Sparc-10, ranging from approximately 5 cpu hours for the 8x8 cases using the perimeter measure to 5 cpu days for the 20x20 cases using the moment measure.

[59]Hill-climbing was used for these cases.

	Grid Size	*Number of Districts*	*Minority Percentage of Population*	*Correlation Between Compactness and Minority Controlled Districts (Somers's d)*	*GERRYMANDER:*
Perimeter	8x10	4	34	0.10 (0.12)	-0.16 (-0.13)
	20x20	16	10	0 (0)	0.00 (0.00)
	20x20	16	25	0.08 (0.16)	
	20x20	16	40	0.25 (0.19)	0.18 (0.16)
Moment of Inertia	8x10	4	34	0.01 (0)	-0.47 (-0.46)
	20x20	16	10	0 (0)	0.00 (0.00)
	20x20	16	25	0.04 (0.06)	
	20x20	16	40	0.13 (0.09)	-0.73 (-0.67)

Table 6: Compactness Effects on Minority Representation with Schelling Distributions.
(500 samples were for each grid/district combination)[60]

The effects in these tables are somewhat more complicated than in tables 3 and 4. Here we see two major effects. First, we see the effect that we would expect to see, given tables 3 and 4 — if the minority party is populous and compact enough the minority party benefits from a compactness rule: They will be able to produce maximal gerrymanders that are more compact, on average than the maximal gerrymanders for the majority party, given the same population distribution (see rows 2–4, 9, and 12–14 in table 5).

On the other hand, remember that if the minority party is weak or dispersed, compactness did not help them very much when districts were created automatically. As I noted in section 4.4.1 the minority would need uncompact districts to capture seats, and hence we see in these cases that compactness harms the minority party: The majority party will be able to produce maximal gerrymanders that look much better than the gerrymanders produced by the minority party.[61]

[60]Hill-climbing was used for these cases.

[61]This onservation is a simplification, since it ignores the boundary condition for minority gerrymandeing. If the minority party population is small enough and scattered enough, they will not capture any districts, even if given

4.4.3 Summary

These results indicate that compactness is not a politically neutral standard. A number of groups stand to gain or lose because of compactness standards. Minorities that are geographically diffuse have good reason to fear compactness standards, which will almost inevitably prevent them from capturing districts. Concentrated minorities that can otherwise rely on having districts hand-tailored to their needs may also find compact standards to their disadvantage. Concentrated minorities who fear majority-controlled gerrymanders, however, may benefit somewhat from compactness standards — although a compact plan still does not *guarantee* representation, and majority-based political groups will, even under compact plans, continue to benefit from the majoritarian bias inherent in districting in general and particularly in geographical districting. It is important to realize however, that the political effects of compactness will depend upon the political institutions that are used to draw districts.

We might expect compactness to have the greatest political effect on racial and ethnic minorities, because, as Table 7 illustrates, these minorities are disproportionately concentrated in large cities.[62] The electoral effects of compactness, will not, however, be limited to these groups; any group that is politically cohesive and geographically concentrated may be affected by mandatory compactness rules.

Census Identification	*Proportion of U.S. Population which Resides in Cities of 100,000 or Greater*

substantial opportunity to gerrymander. In these case, obviously, compactness has no effect on electoral outcomes (see row 5 in table 5 and rows 2 and 6 in table 6).

[62]Race, is in fact, a significant explanatory variable in models of geographic concentration even when other demographic variables are “controlled for”, and is the single most important variable explaining segregation between whites and african-americans. (Alba and Logan 1993)

Black	0.57
American Indian	0.21
Asian	0.53
Hispanic	0.56
White & Other	0.19

Table 7: Proportion of U.S. Population that Resides in Large Cities, by Census Identification.

Source: Abstracted from U.S. Bureau of Census, *Statistical Abstract of the United States 1993.* Washington, DC: Government Printing Office, 1993.

Before closing, I caution that you we should not expect the electoral effects of compactness necessarily to be the same if we measure compactness differently. Other ways of measuring compactness may have different political consequences than those illustrated here. Furthermore, the distributions of real populations in real districts may be lead to effects that are more complex these simple models can capture. The results of this simulation should not be taken as an absolute prediction of *which* groups will benefit from compactness standards so much as a demonstration that compactness methods are not neutral — they can systematically political outcomes.

5. Further Research

The simulation model has three implications for further empirical research on the properties of district compactness.

First, the model shows that compactness effects are nonlinear. Electoral manipulation is much more severely constrained by high compactness than by moderate compactness. Any empirical study of the relationship between gerrymandering and compactness must use models that can accommodate these nonlinearities.

Second, the model shows that the difficulty of drawing compact plans is significantly affected by the shape of the state being divided, as well as by the compactness measure used.

Similarly, differences in population geography may affect the difficulty of drawing compact, equal population plans. Therefore, comparisons of compactness between states are misleading: statistical studies of the electoral effects of compactness should use time-series analysis rather than comparing compactness across states.

Last, the model shows that compactness rules *interact* with the geographical distribution of political groups. The shape of district lines alone is not sufficient to diagnose a gerrymander: A majority which is purposefully attempting to exclude a geographically diffuse political minority from the political arena will want to draw districts that are as compact as possible — whereas the same majority, with the same purpose, facing a geographically concentrated minority, will want to draw noncompact districts. To correctly predict the electoral effects of a set of district lines, you must know the geographical distribution of all the relevant political groups. Geography does matter — but you must interpret it within a political context.

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